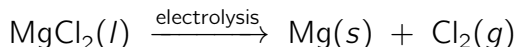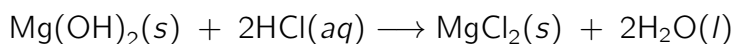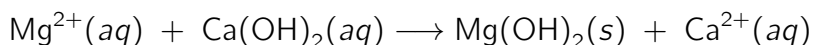# Full Solution

by Jung-Lynn Jonathan Yang

Question

Magnesium metal (a component of alloys used in aircraft and a reducing agent used in the production of uranium, titanium, and other active metals) is isolated from sea water by the following sequence of reactions:

$$Mg^{2+}(aq) \; + \; Ca(OH)_2(aq) \longrightarrow Mg(OH)_2(s) \; + \; Ca^{2+}(aq)$$

$$Mg(OH)_2(s) \; + \; 2HCl(aq) \longrightarrow MgCl_2(s) \; + \; 2H_2O(l)$$

$$MgCl_2(l) \xrightarrow{\text{electrolysis}} Mg(s) \; + \; Cl_2(g)$$

Sea water has a density of 1.026 g/cm^3 and contains 1272 parts per million of magnesium as $Mg^{2+}(aq)$ by mass. What mass, in kilograms, of $Ca(OH)_2$ is required to precipitate 99.9% of the magnesium in 1.00×10^3 L of sea water?

Question source: OpenStax Chemistry 2e

Solution

The concentration in parts per million (ppm) by mass means

$$\text{ppm by mass} = \frac{\text{mass of solute}}{10^6 \text{ mass of solution}}$$

The mass of sea water is

$$m_{\text{sea water}} = (1.00 \times 10^3 \text{ L}) \left(\frac{\text{cm}^3}{10^{-3} \text{ L}} \right) \left(\frac{1.026 \text{ g}}{\text{cm}^3} \right) = 1.03 \times 10^6 \text{ g sea water}$$

Note that cm^3 = mL = 10^{-3} L.
The concentration of Mg^{2+} in sea water is

	$Mg(OH)_2(s)$	$\rightleftharpoons$	$Mg^{2+}(aq)$	+	$2\,OH^-(aq)$
Initial	–		0.0537		?
Change	–		x		$-2x$
Equilibrium	–		$0.0537 - x$		$? - 2x$

1

The equilibrium concentration of Mg^{2+} is the leftover Mg^{2+} in solution. Keep extra digits in the calculator for intermediate calculations because the difference between the initial and equilibrium concentration of Mg^{2+} is very small

$$[Mg^{2+}] = 0.0537 \text{ M} - x = (0.0537 \text{ M})(100\% - 99.9\%)$$
$$x = 0.0536463 \text{ M}$$

Use the K_{sp} expression to calculate the concentration of Ca^{2+} and equate $[OH^-]$ to the equilibrium concentration $? - 2x$ from the ICE table

$$K_{sp} = [Mg^{2+}][OH^-]^2$$
$$[OH^-]^2 = K_{sp}/[Mg^{2+}]$$
$$[OH^-] = \sqrt{K_{sp}/[Mg^{2+}]}$$
$$= \sqrt{(8.9 \times 10^{-12})/(0.0536463)}$$
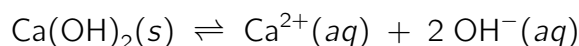$$= 4.07 \times 10^{-4} \text{ M}$$
$$? - 2x = 4.07 \times 10^{-4} \text{ M}$$
$$? = 4.07 \times 10^{-4} \text{ M} + 2x$$
$$= 4.07 \times 10^{-4} \text{ M} + 2(0.0536463 \text{ M})$$
$$= 0.108 \text{ M } OH^- \text{ added}$$

Each mole of $Ca(OH)_2$ has 2 moles of OH^-

$$Ca(OH)_2(s) \rightleftharpoons Ca^{2+}(aq) + 2\, OH^-(aq)$$

$$m_{Ca(OH)_2} = (0.108 \text{ M } OH^-)(1.00 \times 10^3 \text{ L}) \left(\frac{1 \text{ mol } Ca(OH)_2}{2 \text{ mol } OH^-} \right) \left(\frac{74.10 \text{ g } Ca(OH)_2}{\text{mol } Ca(OH)_2} \right)$$
$$= 3.99 \times 10^3 \text{ g } Ca(OH)_2 = \boxed{3.99 \text{ kg } Ca(OH)_2}$$

The difficulty with using $Ca(OH)_2$ to precipitate Mg^{2+} is that $Ca(OH)_2$ has limited solubility in water. The amount of added $[Ca^{2+}]$ and $[OH^-]$ must not exceed the K_{sp} of $Ca(OH)_2$ or else $Ca(OH)_2$ will not dissolve.

$$n_{Ca^{2+}} = (53.8 \text{ M } OH^-)(1.00 \times 10^3 \text{ L}) \left(\frac{1 \text{ mol } Ca^{2+}}{2 \text{ mol } OH^-} \right) = 53.8 \text{ mol } Ca^{2+}$$

$$[Ca^{2+}] = 108 \text{ mol } Ca^{2+}/1.00 \times 10^3 \text{ L} = 0.0538 \text{ M}$$

$$Q_{sp} = [Ca^{2+}][OH^-]$$
$$= (0.0538)(4.07 \times 10^{-4})^2$$
$$= 8.91 \times 10^9 < K_{sp} = 1.3 \times 10^{-6}$$

$Q_{sp} < K_{sp}$, the added $Ca(OH)_2$ will not precipitate.